Heat Capacity

$$C = \lim_{T_i \to T_f} \frac{Q}{T_f - T_i} \equiv \left. \frac{\bar{d}Q}{dT} \right|$$

- In unit of joules per kelvin (J/K)
- It is an extensive quantity (i.e. the larger the mass the larger is the value of *C* since a larger amount of heat is require to heat up the material for 1 degree.)

Specific heat Capacity

$$c = C / m$$

- In unit of joules per kelvin per kg (J/kg·K)
- Intensive quantity, i.e. it’s value remains the same for different amount of mass of the same material. So that the Heat is related to specific heat according to the relation:
- $Q = mc\,\Delta T$

Molar heat capacity

$$c = C / n$$

- *n* is the amount of material measured in unit of mole.
- In unit of joules per kelvin per mole (J/mol·K)
- Intensive quantity, i.e. it's value remains the same for different amount of mass of the same material.

Specific heat capacity : Examples

- **1) How much heat is needed to raise the temperature of a block of copper (0.5 kg) from 0°C to 100° C ?** (for copper, c = 386 J / kg K)
- **2) How much heat is needed to raise the temperature of 0.5 kg of water from 0°C to 100° C?**
- **3)What would be the final temperature of a mixture of 100 g of water at 90°C and 600 g of water at 20°C ?**
- **4)What would be the final temperature if a 2 kg of lead at 200°C are inserted in a container with 10 kg of water at 50°C ?** (for lead, c = 128 J / kg K)

Answers:

- **Answers:**
- 1) Applying the formula: $Q=mc\,\Delta T$
- Q = 386 * 0.5 * 100 = 19300 J or 19.3 kJ
- *Comments: It is important to observe the SI units. The mass is in kg and the heat energy in J. Normally the temperature is converted into K, but because we are taking the difference (or the variation), it doesn't matter what units are used (if kelvin or celsius).*

2)Q = 4186*0.5*100 = 209300 J or 209.3 kJ

Comments: Note that this is more than 10 times the energy needed in the case of copper!

3) This example is also typical and it requires some algebraic skills.
We know that heat flows from the hotter body to the cooler body. Hence, the water at the higher temperature will "loose" heat and the water at the lower initial temperature will "gain" heat. The correct way of describing this situation is by saying that heat will be transferred from the hotter to the cooler water.
We also know that, by conservation of energy, the amount of heat lost will be the same that is gained.

So, let's call the final temperature of the mixture T_f .

The amount of heat that will be transferred from the hotter water is: $Q=mc\,\Delta T=4186*0.1*(90-T_f)$

The amount of heat that will be transferred to the cooler water is: $Q=mc\,\Delta T=4186*0.6*(T_f-20)$

Because these two quantities must be equal, we have an equation:

$4186*0.1*(90-T_f) = 4186*0.6*(T_f-20)$

We need to find T_f:

$418.6 * (90-T_f)=2511.6 * (T_f-20)$

Getting rid of the parenthesis:

$37674-418.6\ T_f=2511.6\ T_f-50232$

$-2930.2\ T_f = -87906$

$T_f = 30°$ C

4) Similar to 3) above,
in this case:
amount of heat transferred
from the lead: $Q=mc\,\Delta T=$
$128*2*(200-T_f)$
amount of heat transferred to
the water: $Q=mc\,\Delta T=$
$4186*10*(T_f-50)$

Equating the two heats:

$$128*2*(200- T_f) = 4186*10*(T_f -50)$$

$$256(200- T_f)=41860(T_f -50)$$

$$51200-256\ T_f = 41860\ T_f - 2093000$$

$$42116\ T_f = 2144200$$

$$T_f = 50.9°$$